

17. Show that $\int_{\gamma} p dx + q dy = 0$, for every $\gamma \sim 0$, in Ω , where $p dx + q dy$ is locally exact in Ω .
18. Discuss the Poisson's formula.
19. State and prove Schwartz's theorem.
20. State and prove Hadamard's theorem.
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NOVEMBER/DECEMBER 2025

23PMA31 — COMPLEX ANALYSIS

Time : Three hours

Maximum : 75 marks



SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Compute $\int_{|z|=1} \frac{e^z}{z} dz$
2. State Cauchy's theorem for a rectangle.
3. Define locally exact.
4. Define multiple connected regions.
5. State Poisson's formula.
6. Write Laplace's equation.
7. State Laurent series.

8. Define Taylor's series.
9. Write Jensen's formula.
10. Define infinite product of complex numbers.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

- (b) State and prove Liouville's theorem.
12. (a) Prove that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω .

Or

- (b) State and prove Rouché's theorem.

13. (a) If u_1 and u_2 are harmonic in a region Ω , show that $\int_{\gamma} u_1^* du_2 - u_2^* du_1 = 0$.

Or

- (b) Discuss Mean-value property.
14. (a) State and prove Weierstrass's theorem.

Or

- (b) Explain in details for Taylor's theorem.
15. (a) Derive the Poisson-Jensen formula.

Or

- (b) Show that $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If $f(z)$ is analytic at z_0 with $f'(z_0) \neq 0$, show that it maps a neighborhood of z_0 conformally and topologically onto a region.